Working Paper

On the Convexity of Direct and Indirect Damage Costs of Extreme Events

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ABSTRACT Understanding the effects of extreme events or disasters on production is crucial for quantifying damage and for identifying options for damage reduction. The latter has got increasing attention owing to the need for adaptation to climate change, since global warming may increase the frequency and intensity of extreme weather events.
The paper provides a micro-foundation of direct and indirect damage costs of extreme events that disturb productive capital. By considering the optimum reconstruction path after an extreme event, we show that both, the direct reconstruction costs and the indirect losses from foregone production, increase convexly if more capital is initially defective. Indirect costs become larger compared to direct costs.

Introduction

A damage function assigns the losses from pollution to the level of pollution. It is a core concept in environmental economics and the integrated assessment of climate change. Usually, damage functions are assumed to be convexly increasing and losses are in terms of utility or output. However, detailed micro-foundations of the concrete structure of damage functions are rare. This is not a purely theoretical question, since, e.g., convexity of damage functions is known to be a crucial prerequisite for Pigouvian taxation to be efficient (e.g. Winrich, 1982). Even importantly, when activities to reduce damage are considered (called averting behavior, protective measures, disaster mitigation or adaptation to climate change in the literature), it is crucial to precisely understand the nature of the externalities (Butler and Maher, 1986; McKitrick and Collinge, 2002).

In this paper we give a precise foundation of damage from (unpredictable) extreme events that affect production. In particular, we have damage from extreme weather events in mind. Some of them are likely to increase in frequency or intensity due to climate change. Understanding the resulting damage costs is crucial for (i) quantifying the costs of climate change and (ii) for identifying parameters that determine the extent of damage and that can be changed by adaptation strategies. To do so, we will both consider the direct costs from reconstructing defective facilities and the indirect costs from the loss of production during reconstruction.

A review on the economics of disasters by Okuyama (2003) collects empirical evidence for a broad variety of economic effects that appear in association with extreme events. Although there are the intrinsic data problems of disaster research, the review nevertheless concludes that there are still important theory gaps. In particular, it discusses the dynamics in the direct aftermath of extremes, for example changes of market equilibrium, capital replacement and the speed of recovery. Already Dacy and Kunrether (1969) discuss possible market failure due to, e.g., rigidities of factor allocation – a thread of the discussion we will take up as well. According to (Albala-Bertrand, 1993) the impact of a natural disaster can be diverse and on physical structure, livestocks and human population. In this paper we concentrate on physical production structure. Hallegatte et al. (2007) develop a model where an extreme event affects all production facilities of an economy equally. The facilities are subsequently reconstructed to the pre-event level. This is in sharp contrast to damage functions in integrated assessment models where damage reduce the amount of capital in the economy. If these models use a macroeconomic production with diminishing returns from capital, this formulation of damage implies that only the least efficient production facilities are affected. Since there is no overall reason for this to happen, just reducing the capital stock strongly underestimates damage. On the other hand, Hallegatte et al. (2007) further assume that all facilities are reconstructed at the same rate. Yet, this may not be efficient since it would be profitable to concentrate reconstruction effort on the more productive facilities first. Our paper will investigate a special case where this simplification is, however, justified. Hallegatte and Dumas (2009) investigate the effects of an extreme event when there is the possibility to replace destroyed capital by more productive capital due to technological progress. In contrast, we concentrate on the case where reconstructing defective capital is always preferable to replacement with new facilities. Gaddis et al. (2007) broadly discuss that the reported damage costs from disasters mostly include only the direct costs, often defined by the replacement costs of the damaged assets, or the insurance value. Thus, substantial indirect costs are usually disregarded. They list very different categories of indirect costs that are partially very difficult to estimate. In our contribution we solely focus on the indirect costs due to loss of production in the affected industry.

Our model considers a single industry with homogeneous output that is affected by an (unpredictable) extreme event, such that a fraction of the capital stock becomes defective. The capital stock is assumed to be decomposed into production facilities with individual production functions, with labor as further input. With a fixed supply, labor is optimally allocated to the facilities to maximize output, obtaining an aggregate production function. By introducing the amount of defective capital as additional variable, we obtain a modified aggregate production function. Its shape depends on the assumptions about the flexibility of allocation of labor. We consider the two extreme cases where labor either is always efficiently allocated, or is rigid at the pre-event level. After an extreme event, the inter-temporally optimum reconstruction path for the industry is determined to minimize total damage costs. Reconstruction requires convexly increasing costs to represent that a faster reconstruction process requires more inputs due to adjustment costs and possible shortage of supply for reconstruction after an extreme event. These direct damage costs add to the loss of production (in comparison to the undisturbed industry), called indirect damage costs of the event.

We find that loss of production at a particular time after an extreme event are a convex function in the amount of defective capital. During reconstruction, these losses accumulate to indirect damage depending on the reconstruction path. The reconstruction time can be shortened at the cost of higher reconstruction expenditures. For the special case of a rigid allocation of labor after an extreme event and quadratic reconstruction costs, both the direct and the indirect damage of an extreme event increase convexly if more capital is initially defective. Yet, indirect damage become more important than direct damage for events with more initially defective capital.

The paper is structured as follows. We begin with a production model of a decomposed capital stock that can be damaged by an extreme event. Since the loss of production depends on how easily labor can be reallocated after an extreme event, two polar cases are derived. Based on that the optimum reconstruction path after an extreme event is determined. Direct and indirect damage costs of an extreme event are computed for a specific case. The results are discussed in the concluding sections.

The Model of Production and Defective Capital

We consider an industry with a capital stock K that is composed of a continuum of $k \in [0, K]$ production facilities with production functions

$$f_k(l_k) = \varepsilon_k l_k^{1-\gamma} \tag{1}$$

with labor input l_k and $0 < \gamma < 1$. The coefficients ε_k represent the production efficiency of the facility $k \in [0, K]$. It is assumed that there is a fixed supply of labor $L = \int_0^K l_k \, \mathrm{d}k$ for the industry. Aggregate output F is then given by the aggregate production function

$$F(K, L) = \max_{l_k, k \in [0,K]} \int_0^K f_k(l_k) \, \mathrm{d}k \,, \tag{2}$$

$$\text{s.t.} \quad L = \int_0^K l_k \, \mathrm{d}k \,. \tag{3}$$

The optimum allocation of labor is determined from the first-order conditions as

$$l_k = \left(\frac{1}{\varepsilon_k} \int_0^K \varepsilon_k \, \mathrm{d}k \right)^{-1} L \,, \tag{4}$$

depending on the profile of efficiency $\varepsilon_k, k \in [0, K]$. Consequently the aggregate production function is a Cobb-Douglas function

$$F(K, L) = \left(\frac{1}{K} \int_0^K \varepsilon_k^{\frac{1}{\gamma}} \, \mathrm{d}k \right)^{\gamma} L^{1-\gamma} K^{\gamma} \,, \tag{5}$$

with decreasing rates of return and constant returns to scale. The factor with the integral is a generalized mean value of the efficiency profile $\varepsilon_k, k \in [0, K]$.

To consider disturbances caused by extreme events, a (temporary) loss of productive capital is represented by z, the stock of defective capital. The difference $K - z$ is the amount of active capital that can be used as an input for production $\widetilde{F}$ during the time after an extreme event. The stock of defective capital is reduced by reconstruction. Obviously, $0 \leq z \leq K$. For the sake of simplicity we assume identical efficiency coefficients $\varepsilon_k = \overline{\varepsilon}$, but discuss it more generally at the end.
Now the question arises how production changes when

$z > 0$ after an extreme event. The results depend on the allocation of labor after the event. We consider two possibilities in the following. In the "flexible" case, labor is instantaneously and efficiently re-allocated to those facilities that are not defective. In the "rigid" case it is assumed that labor cannot be re-allocated. The share of the workforce that is associated with the defective facilities becomes temporarily lost. In practice, allocation of labor is expected to be within both these cases, since options for reallocation are to be expected, but may be limited due to various constraint in the post-disaster situation.
We consider rigid labor first. In this case its allocation is equal to that in Equation (4). Therefore, considering the identical efficiency of all facilities, post-event labor per facility $\tilde{l}_r$ and production $\tilde{f}_r$ become

$$\tilde{l}_r(K,L) = \frac{L}{K}\,;\;\; \tilde{f}_r(K,L) = \bar{\varepsilon}\left(\frac{L}{K}\right)^{\gamma-1}\,. \qquad (6)$$

Here and in the following the subscript r corresponds to the rigid case. The aggregate production function with defective capital z is thus given by

$$\begin{aligned}\tilde{F}_r(K,L,z) &= \int_0^{K-z} \bar{\varepsilon}\tilde{l}_r^{\,1-\gamma}\,\mathrm{d}k \\ &= \left(1-\frac{z}{K}\right)\bar{\varepsilon}L^{1-\gamma}K^{\gamma} \\ &= \left(1-\frac{z}{K}\right)F(K,L). \qquad (7)\end{aligned}$$

Now consider flexible labor. Here the stock of defective capital is accounted for allocation of labor after an extreme event, modifying Equation (4) to

$$\tilde{l}_f = \frac{L}{K-z}\,, \qquad (8)$$

where the subscript f denotes results for the flexible case. This gives the aggregate production function with defective capital

$$\begin{aligned}\tilde{F}_f(K,L,z) &= \int_0^{K-z} \bar{\varepsilon}\left(\frac{L}{K-z}\right)^{1-\gamma}\,\mathrm{d}k \\ &= \left(1-\frac{z}{K}\right)^{\gamma}\bar{\varepsilon}L^{1-\gamma}K^{\gamma} \\ &= \left(1-\frac{z}{K}\right)^{\gamma}F(K,L)\,. \qquad (9)\end{aligned}$$

Thus, depending on the case, the reduced production for a stock of defective capital z either is described by $\tilde{F}_r$ of Equation (7) in case of rigid allocation, or $\tilde{F}_f$ of Equation(9) in case of flexible one. Comparing these two cases with production F for a completely intact capital stock, we can express the loss of production B as

$$B(K,L,z) = F(K,L) - \tilde{F}(K,L,z)\,. \qquad (10)$$

This is

$$B_r(K,L,z) = \frac{z}{K}F(K,L) \qquad (11)$$

for rigid and

$$B_f(K,L,z) = \left[1-\left(1-\frac{z}{K}\right)^{\gamma}\right]F(K,L) \qquad (12)$$

for flexible allocation of labor. Fig. 1 shows the loss of production for both cases. Obviously, defective capital always decreases production, i.e. $B(K,L,z) \geq 0$ as $0 \leq z/K \leq 1$. It is also clear that $B_r(K,L,0) = B_f(K,L,0) = F(K,L)$.
In the rigid case the first and the second derivatives of loss of production are

$$\partial_z B_r = \frac{F(K,L)}{K}\,, \qquad \partial_z^2 B_r = 0\,. \qquad (13)$$

We get a constant increase of loss of production and thus observe a linear dependency between the stock of defective capital and loss of production. In the flexible case the derivatives are

$$\partial_z B_f = \gamma\left(1-\frac{z}{K}\right)^{\gamma-1}\frac{F(K,L)}{K}, \qquad (14)$$

$$\partial_z^2 B_f = \gamma(1-\gamma)\left(1-\frac{z}{K}\right)^{\gamma-2}\frac{F(K,L)}{K^2}\,, \qquad (15)$$

which are both positive. The loss of production is an increasing and convex function of defective capital z.

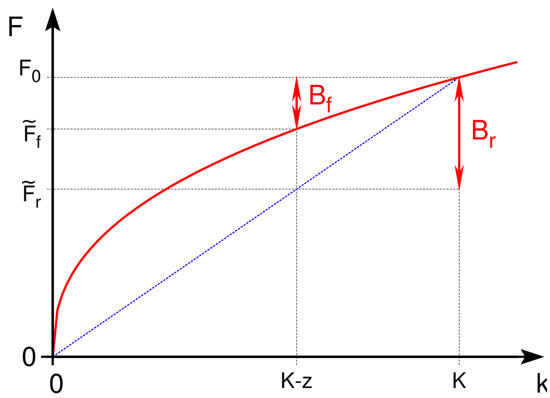

Figure 1: The loss of production, depending on the stock of defective capital z for the case of rigid and flexible allocation of labor after an extreme event.

The Model of Reconstruction After an Extreme Event

When at a point in time an extreme event harms an amount of capital so that it becomes defective, it is the next question how defective capital is reconstructed afterwards. We denote the initial stock of defective capital by $z(0) = z_0$. For the process of repairing we impose some crucial assumptions: (i) repairing the defective capital is cheaper or has higher returns than investing in new one; (ii) reconstruction takes place during a short timescale after the event, such that discounting is not a matter here; (iii) the supply of labor and capital remains constant on that time scale as well; (iv) the market price for the produced output remains constant as well; (v) the occurrence of extreme events is rare compared to the reconstruction time such that there is no new extreme event during recovery time from the last event. The objective of reconstruction is to achieve a minimum (possible) loss of output at the time during reconstruction, taking the costs of reconstruction into account. The decision is about how much reconstruction $R_{(t)}$ is spend at time t to decrease the stock of defective capital z. We suppose that reconstruction services are delivered at convexly increasing cost $c(R)$, measured in industry output as numeraire. This leads to the following problem:

$$\min_{R_{(t)}} \; D(K, L, z_0, R_{(t)}),$$

$$D = \int_0^{t_F} B(K, L, z_{(t)}) + c(R_{(t)}) \, \mathrm{d}t, \tag{16}$$

$$\text{s.t.} \quad \dot{z}_{(t)} = -R_{(t)}\,, \quad z_{(t)} \geq 0\,, \quad R_{(t)} \geq 0. \tag{17}$$

The terminal time t_F is endogenous to the problem and denotes the time where all productive capital is reconstructed $z(t_F) = 0$. The stock of defective capital $z_{(t)}$ is reduced by spending for reconstruction $R_{(t)}$, which reduces the loss of production B at each time.

Next we determine the optimally reconstruction path. The Hamilton is given by

$$H = B(K, L, z) + c(R) - \pi R, \tag{18}$$

where π is the co-state variable. For a strictly convex cost function the Hamilton has a minimum. The first order conditions yield

$$\pi = \partial_R \, c(R)\,, \tag{19}$$

and

$$\dot{\pi} = -\partial_z B(z) = \dot{R} \partial_R^2 c(R)\,. \tag{20}$$

This tells us that we allocate the investment into reconstruction according to its marginal benefit, which is the reduction of loss of production. The transversality condition determines the end of reconstruction t_F where $z(t_F) = 0, R(t_F) = 0$. Using Equations (19) and (20) the dynamics of optimum reconstruction is described by the differential equations

$$\dot{R}^\star = -\frac{\partial_z B(z^\star)}{\partial_R^2 c(R^\star)}\,, \tag{21}$$

$$\dot{z}^\star = -R^\star. \tag{22}$$

Recall that the loss of production B is a convex function of z. Since marginal reconstruction costs increase in R, reconstruction activities are highest directly after an extreme event and decrease afterwards.

For a given optimum reconstruction path $R^\star$, we call the summed costs of reconstruction

$$D_R = \int_0^{t_F} c(R^\star_{(t)}) \, \mathrm{d}t \tag{23}$$

the direct damage D_R of an extreme event and the foregone production compared to the undisturbed production,

$$D_B = \int_0^{t_F} B(K, L, z^\star) \, \mathrm{d}t \tag{24}$$

its indirect damage D_B. The total damage D of an extreme event is the sum of direct and indirect damage, as stated in Equation (16).

As an example we now consider rigid allocation of labor, so that Equation (11) describes the loss of production. Further we take quadratic costs of reconstruction

$$c(R) = aR + \frac{b}{2}R^2 \quad a \in \mathbb{R}, \quad b > 0\,. \tag{25}$$

Here, a and b are constant parameters. According to Equation (21) we have

$$\dot{R}_{(t)} = -\frac{F(K, L)}{Kb} = \mathit{const.}, \quad R_{(t)} \geq 0\,. \tag{26}$$

Now we can solve the two Equations (22) and (26) implying that in the end after a time t_F, the defective capital as well as reconstruction are zero. The solution of the problem is

$$t_F = \sqrt{\frac{2bKz_0}{F(K, L)}}, \tag{27}$$

$$z_{(t)} = z_0 - \sqrt{\frac{2z_0 F(K, L)}{bK}} t + \frac{F(K, L)}{2bK} t^2, \tag{28}$$

$$R_{(t)} = \sqrt{\frac{2z_0 F(K, L)}{bK}} - \frac{F(K, L)}{bK} t, \,. \tag{29}$$

With this, the direct, indirect and total damage caused by an extreme event are

$$D_R = a\, z_0 + \frac{z_0}{3} \sqrt{\frac{2bz_0}{K} F(K, L)}, \tag{30}$$

$$D_B = \frac{z_0}{3} \sqrt{\frac{2bz_0}{K} F(K, L)}, \tag{31}$$

$$D = a\, z_0 + \frac{2z_0}{3} \sqrt{\frac{2bz_0}{K} F(K, L)}, \tag{32}$$

as to see by solving the integrals of Equations (23) and (24). Interestingly, the damage increases more than linearly with the initially defective capital by an exponent of one and a half. Comparing the direct and indirect damage

$$\frac{D_R}{D_B} = 1 + \frac{3a}{\sqrt{\frac{2bz_0}{K} F(K, L)}}, \tag{33}$$

shows that in this example the direct damage costs exceed the indirect damage costs if an optimum reconstruction path is chosen. Further, the ratio increases with more initially defective capital, revealing that the share of indirect damage to total damage costs increases.

Discussion

The analysis defines the reconstruction time as the span between the occurrence of an extreme event and the return of the capital stock to the status before this event. We thus assume that an event itself does not change the long-term production function. This might be a reasonable assumption if there is only limited technological progress and if no further extreme event occurs during reconstruction. Otherwise, persistent changes can result, leading to a quite different assessment of disaster costs.

One should be careful that the total damage costs from an extreme event as determined in this paper are not the same as a damage function in environmental economics. For that, another ingredient is missing, namely the relation of pollution to the strength and frequency of extreme events. If this is a convex correspondence, the damage function will be convex. However, it is likely that distributions of extreme events yield concave results since the probability that a larger part of the capital stock will become defective is lower. For this, it depends whether the convexity of total damage costs depending on the initially defective capital as determined in this paper is so strong that the damage distribution gets a 'fat tail' or not.

The convex damage costs computed in the last section depend on the assumption that the allocation of labor is rigid after an extreme event. Lower damage are to expect, if the more optimistic assumption of an efficient allocation of labor is made. There also might be further short-term market adaptation effects, e.g. when labor costs or demand for industry output changes. Furthermore, other additional costs, as removal expenses, are under circumstances not negligible.

When the total damage costs of an extreme event are a convex function of initially defective capital, an important conclusion for adaptation measures reducing damage can be drawn. We call this argument, which is depicted in Figure 2, "impact aversion". Note that the average damage costs of a "weak" and a "strong" extreme event is higher than the total damage costs of one event with the mean initially defective capital. Thus, when there are measures available (at the same cost) that reduce the initially defective capital, or others alternatively reducing the frequency of extreme events, the former would be preferable. As an example, consider sea surges as extreme events that damage a coastal industry. There are different ways to cope with such risks,

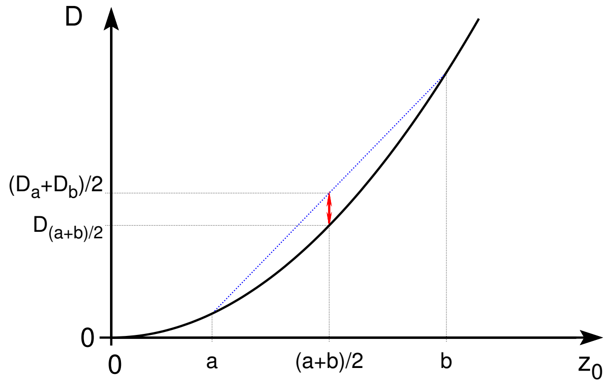

Figure 2: Illustration of impact aversion for convex total damage D depending on the initial level of defective capital z_0.

e.g. building or raising levees, or relocating industry from areas that may be flooded. The former reduces the frequency of surges that affect the industry. Yet, only those floods that are higher than the levee damage industry, but in a similar way as if there were no levee. In contrast, relocation does not reduce the frequency of floods reaching the inland, but decreases the amount of capital that can be damaged. In the light of the "impact aversion" argument, relocation may thus be preferable to structural coastal protection.

Conclusion

The paper shows how a micro-foundation of direct and indirect damage can justify the convexity of total damage from extreme events or disasters (as function of the capital stock that becomes defective due to the disaster). In particular, the optimum reconstruction path reducing defective capital during the aftermath of an event is considered. It is shown that for extreme events resulting in a higher level of defective capital, the share of indirect damage costs to total damage costs increases. This gives valuable hints for disaster mitigation strategies and for adaptation to climate change. The model also clarifies where non-convexities may occur if some simplifying assumptions are invalidated.

One interesting next step would be to relax the assumption of constant efficiency in the capital stock, e.g. by considering vintage capital or facilities with different productivity. This could lead to less damage when an optimum reconstruction path is chosen. Yet, there would be some analytical difficulties since the degree of reconstruction cannot be measured by a scalar. A detailed allocation of reconstruction activities to different parts of the capital stock needs to be determined, instead. We delegate this analysis to a future paper.

However, we hope that the present analysis contributes to the economic theory of disasters, as well as to that of adaptation to climate change.

References

Albala-Bertrand, J. M., 1993. Political economy of large natural disasters. Clarendon Press, Oxford.

Butler, R. V., Maher, M. D., 1986. The control of externalities: Abatement vs. damage prevention. Southern Economic Journal 52, 1088–1102.

Dacy, D. C., Kunrether, H., 1969. The Economics of Natural Disasters: Implications for Federal Policy. New York, NY.

Gaddis, E., Miles, B., Morse, S., Lewis, D., 2007. Full-cost accounting of coastal disasters in the united states: Implications for planning and preparedness. Ecological Economics 63, 307–318.

Hallegatte, S., Dumas, P., 2009. Can natural disasters have positive consequences? investigating the role of embodied technical change. Ecological Economics 68, 777–786.

Hallegatte, S., Hourcade, J. C., Dumas, P., 2007. Why economic dynamics matter in assessing climate change damages: Illustration on extreme events. Ecological Economics 62, 330–340.

McKitrick, R., Collinge, R. A., 2002. The existence and uniqueness of optimal pollution policy in the presence of victim defense measures. Journal of Environmental Economics and Management 44, 106–122.

Okuyama, Y., 2003. Economics of natural disasters - a critical review. Tech. rep., Regional Research Institute - West Virginia University.

Winrich, J. S., 1982. Convexity and corner solutions in the theory of externality. Journal of Environmental Economics and Management 9, 29–41.